# Appendix B

## Transformation to the Plasma Coordinate System

The following is a description of the transformation from the observer's coordinate system to the plasma torus coordinate system used for determining the local plasma state.

(1) The transformation is from the observer's coordinates ($x_0$, $y_0$, $z_0$) to the plasma torus coordinates (L, $\zeta$) where the observer's frame is in heliocentric Jupiter centered coordinates with the positive y axis directed along the sun-Juptier line away from the sun, the positive x axis is directed from east to west, and the z axis positive in the direction of Jupiter north. The Plasma torus frame is defined by the two coordinates M, the M-shell (modified L-shell) of an atoms, and $\zeta$, the distance along the field line from the centrifugal equator to the atom

(2) Local time of each packet (i.e. heliocentric phase angle)

$$\phi = \tan^{-1}\left(\frac{x}{y}\right) \tag{B.1}$$

(3) Initial centrifugal radius:

$$r_0 = \left(x^2 + y^2\right)^{1/2} \tag{B.2}$$

(4) Angle from the orbital equator to the magnetic equator:

$$\alpha = -\eta_{tilt} \cos\left(\lambda - \lambda_0\right) \tag{B.3}$$

(5) Angle from orbital equator to the centrifugal equator:

$$\beta = -\frac{2}{3}\eta_{tilt}\cos(\lambda - \lambda_0) = \frac{2}{3}\alpha \tag{B.4}$$

(6) Angle from centrifugal equator to magnetic equator:

$$\theta = \beta - \alpha = -\frac{1}{3}\alpha \tag{B.5}$$

(7) The L-shell of the packet is determined using an offset tilted dipole magnetic field model with the dipole tilted an angle $\eta_t ilt$ toward $\lambda_{III} = \lambda_0$ and shifted $\delta_D$ towards $\lambda_{III} = \lambda_D$ in the equatorial plane

(8) Offsets in x and y of dipole center from the center of Jupiter

$$\Delta_x = -\delta_D \cos((\lambda - \lambda_D)\sin\phi - \delta_D \sin(\lambda - \lambda_D)\cos\phi \tag{B.6}$$

$$\Delta_y = \delta_D \cos(\lambda - \lambda_D)\cos\phi - \delta_D \sin(\lambda - \lambda_D)\sin\phi \tag{B.7}$$

(9) Centrifugal distance of packet from dipole axis:

$$r_D = \left((x - \Delta x_D)^2 + (y - \Delta y_D)^2\right) \tag{B.8}$$

(10) Angle each point makes with the orbital equator:

$$\gamma_D = tan^{-1}\left(\frac{z}{r_D}\right) \tag{B.9}$$

(11) Magnetic latitude of each packet:

$$\ell = \gamma_D - \alpha \tag{B.10}$$

(12) L-shell of each packet:

$$L = \frac{\left(r_D^2 + z^2\right)}{\cos^2 \ell} \tag{B.11}$$

(13) Distance of the point where the magnetic field line through the packet crosses the centrifugal equator from Jupiter:

$$L' = L\cos^2\theta \tag{B.12}$$

(14) The M-shell is determined from observations of the ribbon. M is the distance from Jupiter in the centrifugal plane of a packet with its position modulated by the observed changes in the distance of the ribbon from Juptier. The effect of a purely East-West electric field is also included. The ribbon is assumed offset from the center of Jupiter a distance $\delta_R$ towards in the direction $\lambda_{III} = \lambda_R$ in the centrifugal plane

(15) Observed offsets in ribbon position:

$$\Delta x_M = [-\delta_R \cos(\lambda - \lambda_R) \sin\phi - \delta_R \sin(\lambda - \lambda_R) \cos\phi] \cos\beta \quad \text{(B.13)}$$

$$\Delta y_M = [\delta_R \cos(\lambda - \lambda_R) \cos\phi - \delta_R \sin(\lambda - \lambda_R) \sin\phi] \cos\beta \quad \text{(B.14)}$$

$$\Delta z_M = -\delta_R \sin\beta \quad \text{(B.15)}$$

(16) Position of each packet relative to the origin of M

$$x_M = -L' \cos\beta \sin\phi + \Delta x_D - \Delta x_M + \epsilon r_0 \quad \text{(B.16)}$$

$$y_M = L' \cos\beta \cos\phi + \Delta y_D - \Delta y_M \quad \text{(B.17)}$$

$$z_M = L' \sin\beta - \Delta z_M \quad \text{(B.18)}$$

(17) M-shell of each packet:

$$M = \left(x_M^2 + y_M^2 + z_M^2\right)^{1/2} \quad \text{(B.19)}$$

(18) The nominal constants used in this analysis are:

$$\eta_{tilt} = 9.8^\circ \quad \text{(B.20)}$$

$$\lambda_0 = 200^\circ \quad \text{(B.21)}$$

$$\delta_D = 0.12 R_J \quad \text{(B.22)}$$

$$\lambda_D = 149^\circ \quad \text{(B.23)}$$

$$\delta_R = 0.59 R_J \quad \text{(B.24)}$$

$$\lambda_R = 149^\circ \quad \text{(B.25)}$$

Variability of these parameters is discussed elsewhere the text.

(19) $\zeta$ is the distance along the field line from the centrifugal equator to the atoms:

$$\zeta = \int_{\theta}^{\ell} \left( (r'(\xi))^2 + (\frac{dr'}{d\xi})^2 \right)^{1/2} d\xi \tag{B.26}$$

where $r'(\xi) = L/\cos^2(\xi)$

(20) The analytic solution to this integral is:

$$\zeta = \left(L\cos^4\xi + 4\cos^2\xi\sin^2\xi\right) \left[ \frac{\tanh^{-1}\left(\frac{\sqrt{6}\sin\xi}{\sqrt{5-3\cos(2\xi)}}\right)\sec\xi}{\sqrt{6}\sqrt{5-3\cos(2\xi)}} + \frac{\tan\xi}{2} \right]\Bigg|_{\alpha}^{\theta} \tag{B.27}$$

(21) Several factors contribute to the electron denisty and temperature at the location of a packet: the M-shell, the distance off the centrifugal axis ($\zeta$), the local time (orbital phase angle $\phi$), and the System-III longitude.

(22) $n_{e,0}$, $T_e$, and $T_i$ in the centrifugal plane are given by the Voyager data (Bagenal 1994) as a function of M. The ribbon is held at a constant distance from Jupiter of M=5.7 $\mathrm{R_J}$.

(23) $T_e$ is asssumed to be constant along field lines.

(24) $n_e$ decreases as a function of the distance along the field line to the centrifugal equator:

$$n_e = n_{e,0} e^{-(zeta/H(L))^2} \tag{B.28}$$

(25) Scale height is determined from the ions in the torus:

$$H = \left(\frac{2kT}{2m^{\star}\Omega R_J^2}\right)^{1/2} \tag{B.29}$$

$$m^{\star} = \frac{m}{1 + Z\left(T_e/T_{\|}\right)} \tag{B.30}$$

with m and Z the masses and charges of torus ions, and $\Omega$ = Jupiter's angular velocity.

(26) Modulation of the Voyager profile is based on observations of the torus from several published sources (Schneider and Trauger 1995; Brown 1995) and observations in this thesis. There are three types of modulation employed:

(a) Local Time Modulation: The observed East/West brightness asymmetry in the torus is indicative of local time variations in the electron denisty and temperature due to the changing magnetic field a packet experiences as it moves through the torus. The correction factor is normalized to western elongation ($\phi = 270°$.

$$C = 1 - \frac{1-R}{2}(1 - \sin\phi)\, n_e = n_{e,0} \times c t_e = t_{e,0} \times c \tag{B.31}$$

where R is the East/West ribbon intensity ratio, with a nominal value of R=0.92.

(b) System-III brightness variation: Based on observations that the ribbon is brighter near $\lambda_I II = 200°$. Electron denisty varies with the square root of the observered intenisty:

$$n_e = n_e \times (1 + A_n \cos(\lambda - \lambda_n)) \tag{B.32}$$

where $A_n$ is the amplitude of the sinusoidal variation and $\lambda_n$ is the longitude of maximum intensity.

(c) System-III scale height variation: Based on observations of the variations in ribbon scale height. Schneider and Trauger (1995) found the variation in parallel ion temperature implied by the observations. An approximate fit to these observations is used here:

$$T_{\|} = -A_T \cos(\lambda - \lambda_T) + T_{avg} \tag{B.33}$$

where $T_{avg}$ is to the average parallel ion temperature, $A_T$ is the amplitude of variation, and $\lambda_T$ is the longitude of minimum scale height. The calculation of the scale height from $T_{\|}$ is described above.

(27) The sodium lifetime follows directly from the knowledge of $n_e$ and $T_e$ as is described elsewhere.